

King Fahd University of Petroleum and Minerals
Dammam Community College

MATH 012 – College Algebra II

Class Test One

Written Exam, Term 092

March 20, 2010

Write your name, ID number and section number.

Name: _____ ID # _____ Sec # _____

This exam consists of Ten questions.

Total _____/45.

Time allowed: One hour and fifteen minutes.

You must show all necessary steps of your solution.

Calculators are not allowed.

This test worth 7.5% of the total marks allocated to this course.

Question	Marks
1	/4
2	/5
3	/5
4	/4
5	/4
6	/5
7	/5
8	/3
9	/6
10	/4
Total marks =	/45

1. If $\frac{\log x}{\log y} = 4$, then find the value of $\frac{\ln \sqrt{x}}{\ln y^3}$ [4 marks]

$$\frac{\ln \sqrt{x}}{\ln y^3} = \frac{\frac{\log \sqrt{x}}{\log e}}{\frac{\log y^3}{\log e}} = \frac{\log \sqrt{x}}{\log e} \cdot \frac{\log e}{\log y^3} = \frac{\log \sqrt{x}}{\log y^3}$$

$$\Rightarrow \frac{\frac{1}{2} \log x}{3 \log y}$$

$$\Rightarrow \frac{1}{6} \cdot \frac{\log x}{\log y} = \frac{1}{6} \cdot 4 = \frac{2}{3}$$

2. Solve the equation [5 marks]

$$\left(\frac{1}{8}\right)^{2x} = 32^{1-x}$$

Change both sides to base 2

$$\frac{1}{8} = 2^{-3} \quad (1 \text{ mark})$$

$$32 = 2^5 \quad (1 \text{ mark})$$

The equation becomes

$$\left((2)^{-3}\right)^{2x} = \left((2)^5\right)^{1-x} \Rightarrow 2^{-6x} = 2^{5-5x} \quad (1 \text{ mark})$$

And so,

$$-6x = 5 - 5x \quad (1 \text{ mark})$$

$$\Rightarrow x = -5 \quad (1 \text{ mark})$$

3. Solve the equation**[5 marks]**

$$\log_2(x-1) + \log_2(x-4) = 2$$

$$\Rightarrow \log_2((x-1)(x-4)) = \log_2 4$$

$$\Rightarrow (x-1)(x-4) = 4$$

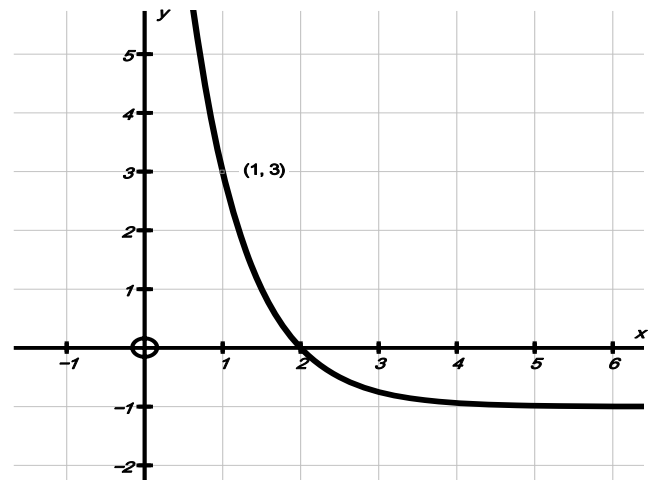
$$\Rightarrow x^2 - 5x = 0$$

$$\Rightarrow x = 0 \text{ or } x = 5$$

But $x = 0$ is invalid since it gives negative arguments for the log functions.

\therefore solution set: $\{5\}$

4. The adjacent figure is the graph of $y = \left(\frac{1}{4}\right)^{x-a} - b$, find the value of $2a+b$

[4 marks]

Horizontal Asymptote $y = -1 \Rightarrow \boxed{b=1}$

At point $(1,3) \Rightarrow 3 = \left(\frac{1}{4}\right)^{1-a} - 1$

$$\Rightarrow 4 = \left(\frac{1}{4}\right)^{1-a} \Rightarrow \left(\frac{1}{4}\right)^{-1} = \left(\frac{1}{4}\right)^{1-a} \Rightarrow -1 = 1-a \Rightarrow \boxed{a=2}$$

$$\therefore 2a+b = 2(2)+1=5$$

5. Given that $y = 2e^{3x-2} + 1$, then find the exact value of y when $x = \ln 3 + \frac{2}{3}$

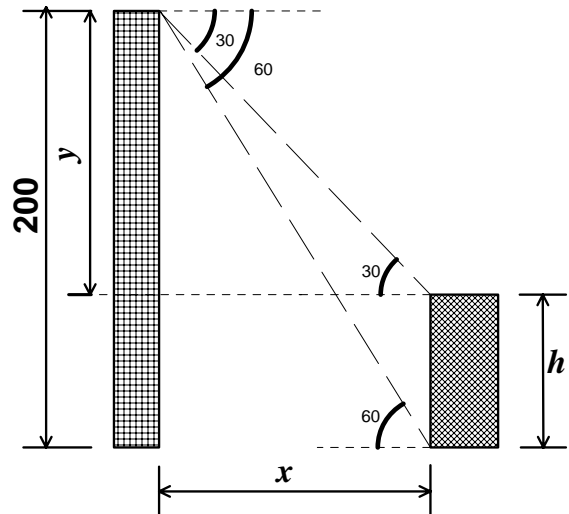
[4 marks]

$$y = 2e^{3\left(\ln 3 + \frac{2}{3}\right) - 2} + 1 = 2e^{3\ln 3 + 2 - 2} + 1$$

$$\Rightarrow y = 2e^{\ln 3^3} + 1 = 2(3^3) + 1 = 2(27) + 1 = 55$$

6. From the top of a tower 200 feet high, the angles of depression of the top and bottom of a building opposite to the tower are observed to be 30° and 60° , respectively. Find the height (h) of the building.

[5 marks]



$$\tan 60^\circ = \frac{200}{x} \Rightarrow x = \frac{200}{\tan 60^\circ} = \frac{200}{\sqrt{3}}$$

$$\tan 30^\circ = \frac{y}{x} \Rightarrow y = x \cdot \tan 30^\circ = \frac{200}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} = \frac{200}{3}$$

$$h = 200 - y = 200 - \frac{200}{3} = \frac{600 - 200}{3} = \frac{400}{3} \text{ feet.}$$

7. The bearing from A to C is $S\ 65^\circ\ E$. The bearing from A to B is $N\ 85^\circ\ E$. The bearing from B to C is $S\ 25^\circ\ W$. A plane flying at 150 mph takes 2.2 hours to go from A to B . Find the distance from A to C .

[5 marks]

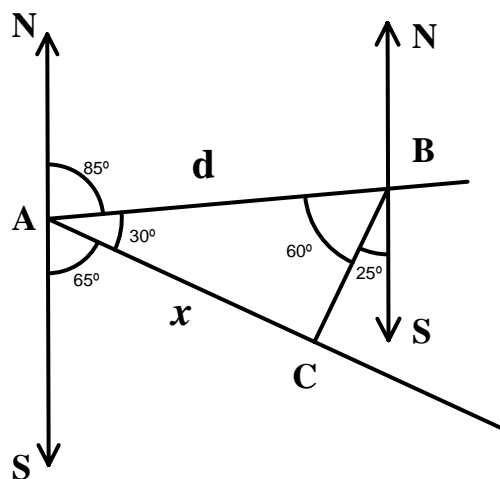
$$d(A, B) = 150\text{ mph} \times 2.2\text{ h} = 330\text{ mi}$$

$$\sin \beta = \frac{x}{d} \Rightarrow x = d \sin \beta$$

$$\Rightarrow x = 330 \sin 60^\circ$$

$$\Rightarrow x = 330 \frac{\sqrt{3}}{2} = 165\sqrt{3}\text{ mi}$$

$$\therefore d(A, C) = 165\sqrt{3}\text{ mi}$$



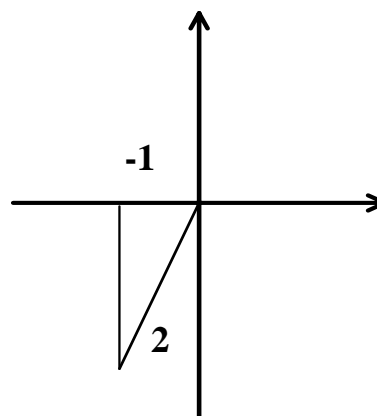
8. Given that $\tan \theta = \sqrt{3}$, where $\theta \in \text{QIII}$. Find $2\sin \theta + 3\cot \theta$.

[3 marks]

$$\sin \theta = \frac{-\sqrt{3}}{2}$$

$$\cot \theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$2\sin \theta + 3\cot \theta = 2\left(-\frac{\sqrt{3}}{2}\right) + 3\left(\frac{\sqrt{3}}{3}\right) = 0.$$



- 9. Find the exact value of this expression: $\sec(-495^\circ) - \csc(-855^\circ) + \cos(510^\circ)$**
[6 marks]

$$\theta = -495^\circ \xrightarrow{\text{Coterminal}} 225^\circ \xrightarrow{\text{Reference}} \theta' = 45^\circ \text{ in } Q III$$

$$\Rightarrow \sec(-495^\circ) = -\sec(45^\circ) = -\sqrt{2}$$

$$\theta = -855^\circ \xrightarrow{\text{Coterminal}} 225^\circ \xrightarrow{\text{Reference}} \theta' = 45^\circ \text{ in } Q III$$

$$\Rightarrow \csc(-855^\circ) = -\csc(45^\circ) = -\sqrt{2}$$

$$\theta = 510^\circ \xrightarrow{\text{Coterminal}} 150^\circ \xrightarrow{\text{Reference}} \theta' = 30^\circ \text{ in } Q II$$

$$\Rightarrow \cos(510^\circ) = -\cos(30^\circ) = -\frac{\sqrt{3}}{2}$$

$$\begin{aligned} \therefore \text{Expression} &= -\sqrt{2} - (-\sqrt{2}) - \frac{\sqrt{3}}{2} \\ &= -\sqrt{2} + \sqrt{2} - \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2} \end{aligned}$$

- 10. Find $\sin \beta$ and $\cos \beta$ given that $\tan \beta = -\frac{2}{5}$ and $\csc \beta > 0$.**

[4 marks]

$$\begin{aligned} \tan \beta &= -\frac{2}{5} \text{ in } QII \text{ \& } QIV \\ \csc \beta &> 0 \text{ in } QI \text{ \& } QII \end{aligned}$$

$$\therefore \beta \text{ in } QII$$

$$r = \sqrt{25 + 4} = \sqrt{29}$$

$$\therefore \sin \beta = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$$

$$\cos \beta = \frac{-5}{\sqrt{29}} = \frac{-5\sqrt{29}}{29}$$

